

## Cusps and a converse to the Ambrosetti-Prodi theorem

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**Abstract.** By the Ambrosetti-Prodi theorem, the map  $F(u) = -\Delta u - f(u)$  between appropriate functional spaces is a global fold. Among the hypotheses, the convexity of the function  $f$  is required. We show in two different ways that convexity is indeed necessary. If  $f$  is not convex, there is a point with at least four preimages under  $F$ . Even more,  $F$  generically admits cusps among its critical points. We present a larger class of nonlinearities  $f$  for which the critical set of  $F$  has cusps. The results are true for Dirichlet, Neumann and periodic boundary conditions, among others.

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