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Geometric singularities and a flow tangent to the Ricci flow

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Abstract. We consider a geometric flow introduced by Gigli and Mantegazza which, in the case of a smooth compact manifold with a smooth metric, is tangential to the Ricci flow almost-everywhere along geodesics. To study spaces with geometric singularities, we consider this flow in the context of a smooth manifold with a rough metric possessing a sufficiently regular heat kernel. On an appropriate non-singular open region, we provide a family of metric tensors evolving in time and provide a regularity theory for this flow in terms of the regularity of the heat kernel.

When the rough metric induces a metric measure space satisfying a Riemannian curvature dimension condition, we demonstrate that the distance induced by the flow is identical to the evolving distance metric defined by Gigli and Mantegazza on appropriate admissible points. Consequently, we demonstrate that a smooth compact manifold with a finite number of geometric conical singularities remains a smooth manifold with a smooth metric away from the cone points for all future times. Moreover, we show that the distance induced by the evolving metric tensor agrees with the flow of $\mathrm{RCD}(K, N)$ spaces defined by Gigli and Mantegazza.

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