Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XIII (2014), 813-858

On the definition and examples of Finsler metrics

MIGUEL ANGEL JAVALOYES AND MIGUEL SÁNCHEZ

To Manuel Barros and Angel Ferrández on their 60^{th} *birthday*

Abstract. For a standard Finsler metric F on a manifold M, the domain is the whole tangent bundle TM and the fundamental tensor g is positive-definite. However, in many cases (for example, for the well-known Kropina and Matsumoto metrics), these two conditions hold in a relaxed form only, namely one has either a *pseudo-Finsler metric* (with arbitrary g) or a *conic Finsler metric* (with domain a "conic" open domain of TM).

Our aim is twofold. First, we want to give an account of quite a few subtleties that appear under such generalizations, say, for *conic pseudo-Finsler metrics* (including, as a preliminary step, the case of *Minkowski conic pseudo-norms* on an affine space). Second, we aim to provide some criteria that determine when a pseudo-Finsler metric F obtained as a general homogeneous combination of Finsler metrics and one-forms is again a Finsler metric – or, more precisely, that the conic domain on which g remains positive-definite. Such a combination generalizes the known (α, β)-metrics in different directions. Remarkably, classical examples of Finsler metrics are reobtained and extended, with explicit computations of their fundamental tensors.

Mathematics Subject Classification (2010): 53C60 (primary); 53C22 (secondary).