

Higher order Glaeser inequalities and optimal regularity of roots of real functions

MARINA GHISI AND MASSIMO GOBBINO

*Dedicated to Professor Sergio Spagnolo
on the occasion of his 70th birthday*

Abstract. We prove a higher order generalization of the Glaeser inequality, according to which one can estimate the first derivative of a function in terms of the function itself and the Hölder constant of its k -th derivative.

We apply these inequalities in order to obtain *pointwise* estimates on the derivative of the $(k + \alpha)$ -th root of a function of class C^k whose derivative of order k is α -Hölder continuous. Thanks to such estimates, we prove that the root is not just absolutely continuous, but its derivative has a higher summability exponent.

Some examples show that our results are optimal.

Mathematics Subject Classification (2010): 26A46 (primary); 26B30, 26A27 (secondary).