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Sharp ill-posedness and well-posedness results for the KdV-Burgers equation: the real line case

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Abstract. We complete the known results on the Cauchy problem in Sobolev spaces for the KdV-Burgers equation by proving that this equation is well-posed in $H^{-1}(\mathbb{R})$ with a solution-map that is analytic from $H^{-1}(\mathbb{R})$ to $C([0,T];H^{-1}(\mathbb{R}))$ whereas it is ill-posed in $H^s(\mathbb{R})$, as soon as $s < -1$, in the sense that the flow-map $u_0 \mapsto u(t)$ cannot be continuous from $H^s(\mathbb{R})$ to even $\mathcal{D}'(\mathbb{R})$ at any fixed $t > 0$ small enough. As far as we know, this is the first result of this type for a dispersive-dissipative equation. The framework we develop here should be useful to prove similar results for other dispersive-dissipative models.

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