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Fine properties of metric space-valued mappings of bounded variation in metric measure spaces

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Abstract. Here we consider two notions of mappings of bounded variation (BV) from a metric measure space into a metric space; one based on relaxations of Newton-Sobolev functions, and the other based on a notion of AM-upper gradients. We show that, when the target metric space is a Banach space, these two notions coincide with comparable energies, but, for more general target metric spaces, the two notions can give different function classes. We then consider the fine properties of BV mappings (based on the AM-upper gradient property), and show that, when the target space is a proper metric space, then for a BV mapping into the target space, codimension-1-almost every point in the jump set of a BV mapping into the proper space has at least two, and at most k_0, jump values associated with it, and that the preimage of balls around these jump values has lower density at least γ at that point. Here, k_0 and γ depend solely on the structural constants associated with the metric measure space, and jump points are points at which the map is not approximately continuous.

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