

Gromov hyperbolic John is quasihyperbolic John. I

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Abstract. In this paper, we introduce the concept of quasihyperbolic John spaces and provide a necessary and sufficient condition for a space to be quasihyperbolic John. Using this criterion, we exhibit a simple proof to show that a John space with a Gromov hyperbolic quasihyperbolization is quasihyperbolic John, quantitatively. This answers in the affirmative an open question proposed by Heinonen (Rev. Mat. Iberoam., 1989) and studied by Gehring *et al.* (Math. Scand., 1989). As a tool, we study the connection between quasihyperbolic John spaces and several geometric conditions, such as the uniformity, the linear local connectedness, the Gehring–Hayman condition, and the ball separation condition.

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