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Counting embeddings of free groups into $\mathrm{SL}_2(\mathbb{Z})$ and its subgroups

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Abstract. We show that, if one selects uniformly independently and identically distributed matrices $A_1, \ldots, A_s \in \mathrm{SL}_2(\mathbb{Z})$ from a ball of large radius X, then with probability at least $1 - X^{-1+o(1)}$ the matrices $A_1, \ldots, A_s$ are free generators of a free subgroup of $\mathrm{SL}_2(\mathbb{Z})$. Furthermore, to show the flexibility of our method, we do a similar counting for matrices from the congruence subgroup $\Gamma_0(Q)$, uniformly with respect to a positive integer $Q \le X$. This improves and generalises a result of E. Fuchs and I. Rivin (2017) which claims that the probability is $1 + o(1)$. We also disprove one of the statements in their work that was used to deduce their claim.

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