

## A linear stochastic model of turbulent cascades and fractional fields

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**Abstract.** Turbulent cascades characterize the transfer of energy injected in a fluid by a random force at large scales towards the small scales. In hydrodynamic turbulence, when the Reynolds number is large, the velocity field of the fluid becomes irregular and the rate of energy dissipation remains bounded from below even if the fluid viscosity tends to zero. A mathematical description of turbulent cascades is a very active research topic since the pioneering work of Kolmogorov in hydrodynamic turbulence and that of Zakharov in wave turbulence. In both cases, these turbulent cascade mechanisms imply power-law behaviors of several statistical quantities such as power spectral densities. For a long time, these cascades were believed to be associated with nonlinear interactions, but recent works have shown that they can also take place in a dynamics governed by a linear equation with a pseudo-differential operator of degree 0. In this spirit, we construct a linear equation that mimics the phenomenology of energy cascades when the external force is a statistically homogeneous and stationary stochastic process. In the Fourier variable, this equation can be seen as a linear transport equation, which corresponds to an operator of degree 0 in physical space. Our results give a complete characterization of the solution: it is smooth at any finite time, and, up to smaller-order corrections, it converges to a fractional Gaussian field at infinite time.

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