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A Łojasiewicz inequality for ALE metrics

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Abstract. We introduce a new functional, inspired by Perelman's λ-functional, adapted to the asymptotically locally Euclidean (ALE) setting and denoted by λ_{ALE}. Its expression includes a boundary term which turns out to be the ADM-mass. We prove that λ_{ALE} is defined and analytic on convenient neighborhoods of Ricci-flat ALE metrics and we show that it is monotonic along the Ricci flow. This for example lets us establish that small perturbations of integrable and stable Ricci-flat ALE metrics with non-negative scalar curvature have non-negative mass. We then introduce a general scheme of proof for a Łojasiewicz-Simon inequality on non-compact manifolds and prove that it applies to λ_{ALE} around Ricci-flat metrics. We moreover obtain an optimal weighted Łojasiewicz exponent for metrics with integrable Ricci-flat deformations.

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