

On symmetries of iterates of rational functions

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Abstract. Let A be a rational function of degree $n \geq 2$. Let us denote by $G(A)$ the group of Möbius transformation σ such that $A \circ \sigma = v_\sigma \circ A$ for some Möbius transformations v_σ , and by $\Sigma(A)$ and $\text{Aut}(A)$ the subgroups of $G(A)$ consisting of σ 's such that $A \circ \sigma = A$ and $A \circ \sigma = \sigma \circ A$, correspondingly. In this paper, we study the sequences of the above groups arising from iterating A . In particular, we show that if A is not conjugate to $z^{\pm n}$, then the orders of the groups $G(A^{\circ k})$, with $k \geq 2$, are finite and uniformly bounded in terms of n only. We also prove a number of results about the groups $\Sigma_\infty(A) = \cup_{k=1}^\infty \Sigma(A^{\circ k})$ and $\text{Aut}_\infty(A) = \cup_{k=1}^\infty \text{Aut}(A^{\circ k})$, which are especially interesting from the dynamical perspective.

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