Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XXVI (2025), 1639-1663

On regularity of maximal distance minimizers in $\mathbb{R}^n$

ALEXEY GORDEEV AND YANA TEPLITSKAYA

Abstract. We study the properties of sets Σ which are the solutions of the maximal distance minimizer problem, *i.e.*, of sets having the minimal length (one-dimensional Hausdorff measure) over the class of closed connected sets $\Sigma \subset \mathbb{R}^n$ satisfying the inequality

$$\max_{y \in M} \operatorname{dist}(y, \Sigma) \leq r$$

for a given compact set $M \subset \mathbb{R}^n$ and some given $r > 0$. Such sets can be considered as the shortest networks of radiating Wi-Fi cables arriving to each customer (for the set M of customers) at a distance at most r.

In this paper we prove that any maximal distance minimizer $\Sigma \subset \mathbb{R}^n$ has at most 3 tangent rays at each point and the angle between any two tangent rays at the same point is at least $2\pi/3$. Moreover, in the plane (for $n = 2$) we show that the number of points with three tangent rays is finite and every maximal distance minimizer is a finite union of simple curves with one-sided tangents continuous from the corresponding side.

All the results are proved for the more general class of local minimizers, *i.e.*, sets which are optimal under perturbation of a neighbourhood of any of their points.

Mathematics Subject Classification (2020): 49Q10 (primary); 49Q20; 49K30; 90C27 (secondary).