

Manifolds with trivial Chern classes I: hyperelliptic manifolds and a question by Severi

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In memory of Mario Baldassarri (1920-1964)

Abstract. We give a negative answer to a question posed by Severi in 1951, whether the Abelian varieties are the only projective manifolds with trivial Chern classes.

By Yau's celebrated result, compact Kähler manifolds with trivial Chern classes must be flat, that is, they belong to the class of hyperelliptic manifolds (quotients T/G of a complex torus T by the free action of a finite group G).

We exhibit simple examples of projective hyperelliptic manifolds which are not Abelian varieties and whose Chern classes are zero not only in integral cohomology, but also in the Chow ring.

We prove, moreover, that the Bagnera-de Franchis manifolds (quotients T/G as above but where the group G is cyclic) have topologically-trivial tangent bundle.

Our results naturally lead to the question of classifying all compact Kähler manifolds with topologically trivial tangent bundle, and to all the counterexamples to Severi's question.

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