On elliptic equations involving surface measures

Marius Müller

Abstract. We show optimal Lipschitz regularity for very weak solutions of the (measure-valued) elliptic PDE $-\mathrm{div}(A(x)\nabla u) = Q \ \mathcal{H}^{n-1} \llcorner \Gamma$ in a smooth domain $\Omega \subset \mathbb{R}^n$. Here, Γ is a $C^{1,\alpha}$-regular hypersurface, $Q \in C^{0,\alpha}$ is a density on Γ, and the coefficient matrix A is symmetric, uniformly elliptic and $W^{1,q}$-regular ($q > n$). We also discuss the optimality of these assumptions on the data. The equation can be understood as a special coupling of two A-harmonic functions with an interface Γ. As such it plays an important role in several free boundary problems, as we shall discuss.