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Reading analytic invariants of parabolic diffeomorphisms from their orbits

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Abstract. In this paper we study germs of diffeomorphisms in the complex plane. We address the following problem: *How to read a diffeomorphism f, knowing one of its orbits $\mathbb{A}$?* We solve this problem for parabolic germs. This is done by associating to the orbit $\mathbb{A}$ a function that we call the *dynamic theta function* $\Theta_{\mathbb{A}}$. We prove that the function $\Theta_{\mathbb{A}}$ is $2\pi\mathrm{i}\mathbb{Z}$-resurgent. We show that one can obtain the sectorial Fatou coordinate as a Laplace-type integral transform of the function $\Theta_{\mathbb{A}}$. This enables one to read the analytic invariants of a diffeomorphism from the theta function of one of its orbits. We also define a closely related *fractal theta function* $\tilde{\Theta}_{\mathbb{A}}$, which is inspired by and generalizes the *geometric zeta function* of a fractal string, and show that it also encodes the analytic invariants of the diffeomorphism.

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