Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XXVI (2025), 1049-1069

The structure of homogeneous Riemannian manifolds with nullity

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Abstract. We find new conditions that the existence of nullity of the curvature tensor of an irreducible homogeneous space $M = G/H$ imposes on the Lie algebra $\mathfrak{g}$ of G and on the Lie algebra $\tilde{\mathfrak{g}}$ of the full isometry group of M. Namely, we prove that there exists a transvection of M in the direction of any element of the nullity, possibly by enlarging the presentation group G. Moreover, we prove that these transvections generate an Abelian ideal of $\tilde{\mathfrak{g}}$. These results constitute a substantial improvement on the structure theory developed in [4]. In addition, we construct examples of homogeneous Riemannian spaces with non-trivial nullity, where G is a non-solvable group, answering a natural open question. Such examples admit (locally homogeneous) compact quotients. In the case of co-nullity 3 we give an explicit description of the isometry group of any homogeneous locally irreducible Riemannian manifold with nullity.

Mathematics Subject Classification (2020): 53C30 (primary); 53C40 (secondary).