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Existence of nonsymmetric logarithmic spiral vortex sheet solutions to the 2D Euler equations

TOMASZ CIEŚLAK, PIOTR KOKOCKI AND WOJCIECH S. OŻAŃSKI

Abstract. We consider solutions of the 2D incompressible Euler equation in the form of $M \geq 1$ concentric logarithmic spirals. We prove the existence of a generic family of spirals that are nonsymmetric in the sense that the angles of the individual spirals are not uniformly distributed over the unit circle. Namely, we show that if $M = 2$ or $M \geq 3$ is an odd integer such that certain non-degeneracy conditions hold, then, for each $n \in \{1, 2\}$, there exists a logarithmic spiral with M branches of relative angles arbitrarily close to $\overline{\theta}_k = kn\pi/M$ for $k = 0, 1, \ldots, M-1$, which include halves of the angles of the Alexander spirals. We show that the non-degeneracy conditions are satisfied if $M \in \{2, 3, 5, 7, 9\}$, and that the conditions hold for all odd $M > 9$ given a certain gradient matrix is invertible, which appears to be true by numerical computations.

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