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Optimal Gevrey regularity for certain sums of squares in two variables

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Abstract. For q, a integers such that $a \geq 1$ and $1 < q$, taking (x, y) in U, where U is a neighborhood of the origin in $\mathbb{R}^2$, we consider the operator

$$P = D_x^2 + x^{2(q-1)} D_y^2 + y^{2a} D_y^2.$$

Slightly modifying the method of proof of [9] one can see that it is Gevrey s_0 hypoelliptic, where $s_0^{-1} = 1 - a^{-1}(q-1)q^{-1}$. Here we show that this value is optimal, *i.e.*, that there are solutions u to $Pu = f$ with f more regular than G^{s_0}, the Gevreyclass of order s_0, that are not better than Gevrey s_0.

The above operator reduces to the Métivier operator [24] when $a = 1$ and $q = 2$. We give a description of the characteristic manifold of the operator and of its relation with the Treves conjecture on the analytic hypoellipticity for sums of squares.

A result of this type is an essential step to prove that there is no analytic hypoellipticity when the characteristic variety is not a symplectic manifold.

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