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Bounded cohomology of finitely presented groups: vanishing, non-vanishing, and computability

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Abstract. We provide new computations in bounded cohomology:

A group is boundedly acyclic if its bounded cohomology with trivial real coefficients is zero in all positive degrees. We show that there exists a continuum of finitely generated non-amenable boundedly acyclic groups and construct a finitely presented non-amenable boundedly acyclic group.

On the other hand, we construct a continuum of finitely generated groups, whose bounded cohomology has uncountable dimension in all degrees greater than or equal to 2, and a concrete finitely presented one.

Countable non-amenable groups with these two extreme properties were previously known to exist, but these constitute the first finitely generated/finitely presented examples.

Finally, we show that various algorithmic problems on bounded cohomology are undecidable.

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