Minimizing under relaxed symmetry constraints: triple and N-junctions

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Abstract. We consider a non-negative potential $W : \mathbb{R}^2 \to \mathbb{R}$ invariant under the action of the rotation group C_N of the regular polygon with N sides, $N \geq 3$. We assume that W has N nondegenerate zeros and prove the existence of an N-junction solution $U : \mathbb{R}^2 \to \mathbb{R}^2$ to the vector Allen-Cahn equation. The structure of U is such that $\mathbb{R}^2$ is divided in N approximate sectors associated to the N zeros of W and, in each sector, U converges to the corresponding zero of W. Assuming only C_N invariance requires a new approach with respect to the one utilized for W invariant under the whole symmetry group Z_N of the regular N-gon. Our proof is variational and is based on sharp lower and upper bounds for the energy of minimizers and on a special pointwise estimate for vector minimizers.

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