

Multiplicity and degree relative to a set

VINCENT GRANDJEAN AND MARIA MICHALSKA

Abstract. The multiplicity of an analytic function germ f relative to a semi-analytic subset germ $(S, 0)$ of $(\mathbb{R}^n, 0)$ is the largest exponent q such that the inequality $|f(x)| \leq M\|x\|^q$ holds on $(S, 0)$ for some constant M . We show the existence of a family of semialgebraic curves $\{\Gamma_d\}_{d \in \mathbb{N}}$ determined only by $(S, 0)$ such that the multiplicity of any polynomial of degree d relative to $(S, 0)$ is equal to its multiplicity relative to Γ_d . Moreover, a semianalytic family $(S_t)_{t \in \mathbb{R}^m}$ given by inequalities $f_i - t_i g_i \geq 0$ for $i = 1, \dots, m$ admits a stratification of the parameter space \mathbb{R}^m such that on each component of the top-dimensional stratum the relative multiplicity function on \mathcal{O}_n does not change.

One can apply properties of relative multiplicity to obtain analogous results for the degree of a polynomial f relative to a semialgebraic set S , defined as the smallest exponent r such that the inequality $|f(x)| \leq D\|x\|^r$ holds on the germ (S, ∞) , for a constant D .

Mathematics Subject Classification (2020): 14P15 (primary); 32S45, 32B20 (secondary).