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On the classification of ancient solutions to curvature flows on the sphere

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Abstract. We consider the evolution of hypersurfaces on the unit sphere $\mathbb{S}^{n+1}$ by smooth functions of the Weingarten map. We introduce the notion of "quasi-ancient" solutions for flows that do not admit nontrivial, convex, ancient solutions. Such solutions are somewhat analogous to ancient solutions for flows, such as the mean curvature flow, or 1-homogeneous flows. The techniques presented here allow us to prove that any convex, quasi-ancient solution of a curvature flow which satisfies a backwards in time uniform bound on mean curvature must be stationary or a family of shrinking geodesic spheres. The main tools are geometric, employing the maximum principle, a rigidity result in the sphere and an Aleksandrov reflection argument. We emphasize that no homogeneity or convexity/concavity restrictions are placed on the speed, though we do also offer a short classification proof for several such restricted cases.

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