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Rigidity of ε-harmonic maps of low degree

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Abstract. In 1981, Sacks and Uhlenbeck introduced their famous α-energy as a way to approximate the Dirichlet energy and produce harmonic maps from surfaces into Riemannian manifolds. However, the second and third authors together with Malchiodi [11, 12] showed that for maps between two-spheres this method does not capture every harmonic map. They established a gap theorem for α-harmonic maps of degree zero and also showed that below a certain energy bound α-harmonic maps of degree one are rotations. We establish similar results for ε-harmonic maps $u_\varepsilon \colon S^2 \to S^2$, which are critical points of the ε-energy introduced by the second author in [9]. In particular, we similarly show that ε-harmonic maps of degree zero with energy below 8π are constant and that maps of degree ± 1 with energy below 12π are of the form Rx with $R \in O(3)$. Moreover, we construct nontrivial ε-harmonic maps of degree zero with energy $> 8\pi$.

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