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Eigenvalue bounds for the Paneitz operator and its associated third-order boundary operator on locally conformally flat manifolds

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Abstract. In this paper we study bounds for the first eigenvalue of the Paneitz operator P and its associated third-order boundary operator B^3 on four-manifolds. We restrict to orientable, simply connected, locally conformally flat manifolds that have at most two umbilic boundary components. The proof is based on showing that under the hypotheses of the main theorems, the considered manifolds are confomally equivalent to canonical models. This equivalence is proved by showing the injectivity of suitable developing maps. Then the bounds on the eigenvalues are obtained through explicit computations on the canonical models and its connections with the classes of manifolds that we are considering. In particular, we give an explicit bound for a 4-dimensional annulus with a radially symmetric metric. The fact that P and B^3 are conformal in four dimensions is key in the proof.

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