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Towards optimal transport for quantum densities

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Abstract. An analogue of the quadratic Wasserstein (or Monge-Kantorovich) distance between Borel probability measures on $\mathbb{R}^d$ has been defined in [14] for density operators on $L^2(\mathbb{R}^d)$, and used to estimate the convergence rate of various asymptotic theories in the context of quantum mechanics. The present work proves a Kantorovich-type duality theorem for this quantum variant of the Monge-Kantorovich or Wasserstein distance, and discusses the structure of optimal quantum couplings. Specifically, we prove that, under some boundedness and constraint hypothesis on the Kantorovich potentials, optimal quantum couplings involve a gradient-type structure similar in the quantum paradigm to the Brenier transport map. On the contrary, when the two quantum densities have finite rank, the structure involved by the optimal coupling has, in general, no classical counterpart.

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