Sobolev metrics on spaces of manifold-valued curves

MARTIN BAUER, CY MAOR AND PETER W. MICHOR

Abstract. We study completeness properties of reparametrization-invariant Sobolev metrics of order $n \geq 2$ on the space of open and closed immersed curves in a manifold. In particular, for several important classes of metrics, we show that Sobolev immersions are metrically and geodesically complete (thus the geodesic equation is globally well-posed). These results were previously known only for closed curves with values in Euclidean space. For the class of constant-coefficient Sobolev metrics on open curves, we show that they are metrically incomplete, and that this incompleteness only arises from curves that vanish completely (unlike "local" failures that occur in lower-order metrics).