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On shrinking targets and self-returning points

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Abstract. We consider the set $\mathcal{R}_{\text{io}}$ of points returning infinitely many times to a sequence of shrinking targets around themselves. Under additional assumptions we improve Boshernitzan's pioneering result on the speed of recurrence. In the case of the doubling map as well as for some linear maps on the d-dimensional torus, we even obtain a dichotomy condition for $\mathcal{R}_{\text{io}}$ to have measure-zero or one. Moreover, we study the set of points eventually always returning and prove an analogue of Boshernitzan's result in similar generality.

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