

Willmore obstacle problems under Dirichlet boundary conditions

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Abstract. We consider obstacle problems under Dirichlet boundary conditions for the Euler's elastica functional in the class of one-dimensional graphs over the real axis and for the Willmore functional in the class of surfaces of revolution. We prove the existence of minimisers of the obstacle problems under the assumption that the elastic or, respectively, the Willmore energy, under the unilateral constraint, is below a universal bound. We address the question whether such bounds are necessary in order to ensure the solvability of the obstacle problems. Moreover, we give several instructive examples of obstacles for which minimisers exist.

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