

## Non-orientable slice surfaces and inscribed rectangles

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**Abstract.** We discuss differences between genera of smooth and locally-flat non-orientable surfaces in the 4-ball with boundary a given torus knot or 2-bridge knot. In particular, we establish that a result by Batson on the smooth non-orientable 4-genus of torus knots does not hold in the locally-flat category. We further show that certain families of torus knots are not boundaries of embedded Möbius bands in the 4-ball and other 4-manifolds.

Our investigation of non-orientable surfaces whose boundary is a given torus knot is motivated by our approach to unify the proof of the existence of inscribed squares and of inscribed rectangles with aspect ratio  $\sqrt{3}$  in Jordan curves with a regularity condition. This generalizes a result by Hugelmeyer for smooth Jordan curves.

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