Klt varieties of general type with small volume

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Abstract. By Hacon-McKernan-Xu, there is a positive lower bound in each dimension for the volume of all klt varieties with ample canonical class. We show that these bounds must go to zero extremely fast as the dimension increases, by constructing a klt n-fold with ample canonical class whose volume is less than $1/2^{2^n}$. These examples should be close to optimal.

We also construct, for every n, a klt Fano variety of dimension n such that the space of sections $H^0(X, -mK_X)$ is zero for all $1 \leq m < b$ with b roughly 2^{2^n}. Here again there is some bound in each dimension, by Birkar's theorem on boundedness of complements, and we are showing that the bound must increase extremely fast with the dimension.