Regular-singular connections on relative complex schemes

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Abstract. Deligne's celebrated "Riemann-Hilbert correspondence" appearing in [9] relates representations of the fundamental group of a smooth complex algebraic variety and regular-singular integrable connections. In this work, we show how to arrive at a similar statement in the case of a smooth scheme X over the spectrum of a ring $R = \mathbb{C}[\![t_1, \ldots, t_r]\!]/I$. On one side of the correspondence we have representations on R-modules of the fundamental group of the special fibre, and on the other we have certain integrable R-connections admitting logarithmic models. The correspondence is then applied to give explicit examples of differential Galois groups of $\mathbb{C}[\![t]\!]$-connections.