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Filippov's theorem for mutational inclusions in a metric space

HÉLÈNE FRANKOWSKA AND THOMAS LORENZ

Abstract. This article is devoted to an extension of the celebrated Filippov theorem to the metric space setting. We deal with fairly general metric spaces, where derivatives of time-dependent functions are replaced by mutations and solutions of differential equations/inclusions are mutational primitives of (time-dependent) maps of transitions. As an example of application we discuss measure-valued solutions to a controlled transport equation and state the Filippov theorem in this context. We also show that whenever a transport equation is generated by Lipschitz vector fields its classical weak solutions coincide with its mutational solutions. Our abstract setting applies as well to systems on the space of nonempty compact subsets of $\mathbb{R}^n$ endowed with the Pompeiu-Hausdorff distance and to continuity equations/inclusions on Wasserstein spaces of Borel probability measures.

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