Characterization of affine surfaces with a torus action by their automorphism groups

ALVARO LIENDO, ANDRIY REGETA AND CHRISTIAN URECH

Abstract. In this paper we prove that if two normal affine surfaces S and S' have isomorphic automophism groups, then every connected algebraic group acting regularly and faithfully on S acts also regularly and faithfully on S'. Moreover, if S is non-toric, we show that the dynamical type of a 1-torus action is preserved in presence of an additive group action. We also show that complex affine toric surfaces are determined by the abstract group structure of their regular automorphism groups in the category of complex normal affine surfaces using properties of the Cremona group. As a generalization to arbitrary dimensions, we show that complex affine toric varieties, with the exception of the algebraic torus, are uniquely determined in the category of complex affine normal varieties by their automorphism groups seen as ind-groups.

Mathematics Subject Classification (2020): 14R20 (primary); 14J50, 14M25, 22F50, 14E07 (secondary).