Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XXIV (2023), 141-172

L^1 solutions to parabolic Keller-Segel systems involving arbitrary superlinear degradation

MICHAEL WINKLER

Abstract. The chemotaxis system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u\nabla v) + f(u) \\ v_t = \Delta v - v + u, \end{cases}$$

is considered under homogeneous Neumann boundary conditions and with nonnegative integrable initial data in smoothly bounded n-dimensional domains with $n \geq 2$, where $f \in C^1([0,\infty))$ is supposed to generalize standard choices of logistic-type reproduction and degradation, as obtained on letting $f(s) = \rho s - \mu s^\alpha$ for $s \geq 0$, with $\rho \geq 0$, $\mu > 0$ and $\alpha > 1$.

Previous results in this direction have identified various conditions on the growth of $-f(s)$ as $s \to \infty$ which ensure global existence of either classical or generalized solutions, but all precedents acting in frameworks of at least integrable solutions seem to rely on considerably strong superlinear degradation for such conclusions, with the apparently farthest-reaching result after all requiring that in the above setting the inequality $\alpha > \frac{2n+4}{n+4}$ holds.

In the present work it is shown that besides the basic requirement that $f(0)$ be nonnegative, the mere assumption

$$\frac{f(s)}{s} \to -\infty \qquad \text{as } s \to \infty \tag{$\star$}$$

on superlinearity of degradation is sufficient to allow for the construction of globally defined nonnegative and integrable functions u and v such that (u, v) solves an associated initial-boundary value problem in an appropriately generalized sense. In particular, this indicates that already the mild condition $(\star)$ rules out the emergence of persistent Dirac-type singularities, known as a characteristic feature of unperturbed Keller-Segel systems corresponding to the choice $f \equiv 0$.

Mathematics Subject Classification (2020): 35D99 (primary); 92C17, 35Q92, 35K55 (secondary).