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A noncommutative calculus on the cyclic dual of Ext

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Abstract. We show that if the cochain complex computing Ext groups (in the category of modules over Hopf algebroids) admits a cocyclic structure, then the noncommutative Cartan calculus structure on Tor over Ext dualises in a cyclic sense to a calculus on Coext over Cotor. More precisely, the cyclic duals of the chain respectively cochain spaces computing the two classical derived functors lead to complexes that compute the more exotic ones, giving a cyclic opposite module over an operad with multiplication that induce operations such as a Lie derivative, a cap product (or contraction), and a (cyclic) differential, along with higher homotopy operators defining a noncommutative Cartan calculus up to homotopy. In particular, this allows to recover the classical Cartan calculus from differential geometry or the Chevalley-Eilenberg calculus for Lie(-Rinehart) algebras without any finiteness conditions or the use of topological tensor products.

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