The L^1-contraction principle in optimal transport

MATT JACOBS, INWON KIM AND JIAJUN TONG

Abstract. In this work we use the JKO scheme to approximate a general class of diffusion problems generated by Darcy's law. Although the scheme is now classical, if the energy density is spatially inhomogeneous or irregular, many standard methods cannot be applied to establish convergence to the continuum limit. To overcome these difficulties, we analyze the scheme through its dual problem and establish a novel L^1-contraction principle for the density variable. As a consequence, we obtain a new L^1-equicontinuity property for the discrete solutions, which gives rise to strong L^1-convergence of the densities to a weak solution of the continuum equation. Notably, the contraction principle relies only on the existence of an optimal transport map and the convexity structure of the energy. As a result, the principle holds in a very general setting, and opens the door to using optimal-transport-based variational schemes to study a larger class of non-linear inhomogeneous parabolic equations.

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