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Heat kernel estimates for general symmetric pure jump Dirichlet forms

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Abstract. In this paper we consider the following symmetric non-local Dirichlet forms of pure jump type on a metric measure space (M, d, μ):

$$\mathcal{E}(f, g) = \int_{M\times M} (f(x) - f(y))(g(x) - g(y))\, J(dx, dy),$$

where $J(dx, dy)$ is a symmetric Radon measure on $M \times M \setminus \text{diag}$ that may have different scalings for small jumps and large jumps. Under a general volume doubling condition on (M, d, μ) and some mild quantitative assumptions on $J(dx, dy)$ that are allowed to have light tails of polynomial decay at infinity, we establish stability results for two-sided heat kernel estimates as well as heat kernel upper bound estimates in terms of jumping kernel bounds, the cut-off Sobolev inequalities, and the Faber-Krahn inequalities (respectively, the Poincaré inequalities). We also give stable characterizations of the corresponding parabolic Harnack inequalities.

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