

The 0-fractional perimeter between fractional perimeters and Riesz potentials

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Abstract. This paper provides a unified point of view on fractional perimeters and Riesz potentials. Denoting by H^σ - for $\sigma \in (0, 1)$ - the σ -fractional perimeter and by J^σ - for $\sigma \in (-d, 0)$ - the σ -Riesz energies acting on characteristic functions, we prove that both functionals can be seen as limits of renormalized self-attractive energies as well as limits of repulsive interactions between a set and its complement.

We also show that the functionals H^σ and J^σ , up to a suitable additive renormalization diverging when $\sigma \rightarrow 0$, belong to a continuous one-parameter family of functionals, which for $\sigma = 0$ gives back a new object we refer to as *0-fractional perimeter*. All the convergence results with respect to the parameter σ and to the renormalization procedures are obtained in the framework of Γ -convergence. As a byproduct of our analysis, we obtain the isoperimetric inequality for the 0-fractional perimeter.

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