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Non-convex functionals penalizing simultaneous oscillations along independent directions: rigidity estimates

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Abstract. We study a family of non-convex functionals $\{\mathcal{E}\}$ on the space of measurable functions $u : \Omega_1 \times \Omega_2 \subset \mathbf{R}^{n_1} \times \mathbf{R}^{n_2} \to \mathbf{R}$. These functionals vanish on the non-convex subset $S(\Omega_1 \times \Omega_2)$ formed by functions of the form $u(x_1, x_2) = u_1(x_1)$ or $u(x_1, x_2) = u_2(x_2)$. We investigate under which conditions the converse implication "$\mathcal{E}(u) = 0 \Rightarrow u \in S(\Omega_1 \times \Omega_2)$" holds. In particular, we show that the answer depends strongly on the smoothness of u. We also obtain quantitative versions of this implication by proving that (at least for some parameters) $\mathcal{E}(u)$ controls in a strong sense the distance of u to $S(\Omega_1 \times \Omega_2)$.

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