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C_p estimates for rough homogeneous singular integrals and sparse forms

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Abstract. We consider Coifman–Fefferman inequalities for rough homogeneous singular integrals T_Ω and C_p weights. Recently, the second author, Pérez, Rivera-Ríos and the third author showed that

$$\|T_\Omega\|_{L^p(w)} \le C_{p,\Omega,w}\|Mf\|_{L^p(w)}$$

for every $0 < p < \infty$ and every $w \in A_\infty$. Our first goal is to generalize this result for every $w \in C_q$ where $q > \max\{1, p\}$ without using extrapolation theory. Although the bounds we prove are new even in a qualitative sense, we also give the quantitative bound with respect to the C_q characteristic. Our techniques rely on recent advances in sparse domination theory and we actually prove most of our estimates for sparse forms.

Our second goal is to continue the structural analysis of C_p classes. We consider some weak self-improving properties of C_p weights and weak and dyadic C_p classes. We also revisit and generalize a counterexample by Kahanpää and Mejlbro who showed that $C_p \setminus \bigcup_{q>p} C_q \neq \emptyset$. We combine their construction with techniques of Lerner to define an explicit weight class $\widetilde{C}_p$ such that $\bigcup_{q>p} C_q \subsetneq \widetilde{C}_p \subsetneq C_p$ and every $w \in \widetilde{C}_p$ satisfies Muckenhoupt's C_p conjecture. In particular, we give a different, self-contained proof for the fact that the $C_{p+\varepsilon}$ condition is not necessary for the Coifman–Fefferman inequality, and our ideas allow us to consider also dimensions higher than 1.

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