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On the rationality problem for forms of moduli spaces of stable marked curves of positive genus

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Abstract. Let $M_{g,n}$ (respectively, $\overline{M}_{g,n}$) be the moduli space of smooth (respectively stable) curves of genus g with n marked points. Over the field of complex numbers, it is a classical problem in algebraic geometry to determine whether or not $M_{g,n}$ (or equivalently, $\overline{M}_{g,n}$) is a rational variety. Theorems of J. Harris, D. Mumford, D. Eisenbud and G. Farkas assert that $M_{g,n}$ is not even unirational for any $n \geqslant 0$ if $g \geqslant 22$. Moreover, P. Belorousski and A. Logan showed that $M_{g,n}$ is unirational for only finitely many pairs (g, n) with $g \geqslant 1$. Finding the precise range of pairs (g, n), where $M_{g,n}$ is rational, stably rational or unirational, is a problem of ongoing interest.

In this paper we address the rationality problem for twisted forms of $\overline{M}_{g,n}$ defined over an arbitrary field F of characteristic $\neq 2$. We show that all F-forms of $\overline{M}_{g,n}$ are stably rational for $g = 1$ and $3 \leqslant n \leqslant 4$, for $g = 2$ and $2 \leqslant n \leqslant 3$, for $g = 3$ and $1 \leqslant n \leqslant 14$, $g = 4$ and $1 \leqslant n \leqslant 9$, and for $g = 5$ and $1 \leqslant n \leqslant 12$.

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