

Hénon equation involving nearly critical Sobolev exponent in a general domain

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Abstract. In this paper we are concerned with the following Hénon problem

$$\begin{cases} -\Delta u = |x|^\alpha u^{2^*-1-\varepsilon} & u > 0 & \text{in } \Omega \\ u = 0 & & \text{on } \partial\Omega, \end{cases}$$

where $N \geq 4$, $2^* = \frac{2N}{N-2}$, $\alpha > 0$, ε is a small positive parameter, Ω is a smooth bounded domain in \mathbb{R}^N and $0 \in \Omega$. Most of the previous works for the Hénon problems were investigated in special domains, such as balls and annuli. In this paper we will study the case when Ω is a more general domain, which does not satisfy symmetry any more. We first investigate the necessary condition on the location of the blow-up point for the peak solution to the above the Hénon problem. Then, we prove that, as $\varepsilon \rightarrow 0$, the above problem has a positive solution with multiple bubbles under a suitable condition on the geometry of Ω .

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