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On Mean Value formulas for solutions to second order linear PDEs

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Abstract. In this paper we give a general proof of Mean Value formulas for solutions to second order linear PDEs, only based on the local properties of their fundamental solution Γ. Our proof requires a kind of pointwise vanishing integral condition for the intrinsic gradient of Γ. Combining our Mean Value formulas with a "descent method" due to Kuptsov, we obtain formulas with improved kernels. As an application, we implement our general results to heat operators on stratified Lie groups and to Kolmogorov operators.

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