On the Schottky problem for genus-five Jacobians with a vanishing theta-null

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Abstract. We give a solution to the weak Schottky problem for genus-five Jacobians with a vanishing theta-null, answering a question of Grushevsky and Salvati Manni. More precisely, we show that if a principally polarized Abelian variety of dimension five has a vanishing theta-null with a quadric tangent cone of rank at most three, then it is in the Jacobian locus, up to extra irreducible components. We employ a degeneration argument, together with a study of the ramification loci for the Gauss map of a theta divisor.