

Stratification and averaging for exponential sums: bilinear forms with generalized Kloosterman sums

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Abstract. We introduce a new comparison principle for exponential sums over finite fields in order to study “sum-product” sheaves that arise in the study of general bilinear forms with coefficients given by trace functions modulo a prime q . When these functions are hyper-Kloosterman sums with characters, we succeed in establishing cases of this principle that lead to non-trivial bounds below the Pólya-Vinogradov range. This property is proved by a subtle interplay between étale cohomology in its algebraic and diophantine incarnations. We give a first application of our bilinear estimates concerning the first moment of a family of L -functions of degree 3.

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