Optimal estimates for the triple junction function and other surprising aspects of the area functional

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Dedicated to Francesco

Abstract. We consider the relaxed area functional for vector valued maps and its exact value on the triple junction function $u : B_1(O) \to \mathbb{R}^2$, a specific function which represents the first example of map whose graph area shows nonlocal effects. This is a map taking only three different values $\alpha, \beta, \gamma \in \mathbb{R}^2$ in three equal circular sectors of the unit radius ball $B_1(O)$. We prove a conjecture due to G. Bellettini and M. Paolini asserting that the recovery sequence provided in [5] (and the corresponding upper bound for the relaxed area functional of the map u) is optimal. At the same time, we show by means of a counterexample that such construction is not optimal if we consider different domains than $B_1(O)$, which still contain the same discontinuity set of u in $B_1(O)$. Such domains are obtained from $B_1(O)$ erasing part of interior of the sectors where u is constant.