

Level curve portraits of rational inner functions

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Abstract. We analyze the behavior of rational inner functions on the unit bidisk near singularities on the distinguished boundary \mathbb{T}^2 using level sets. We show that the unimodular level sets of a rational inner function ϕ can be parametrized with analytic curves and connect the behavior of these analytic curves to that of the zero set of ϕ . We apply these results to obtain a detailed description of the fine numerical stability of ϕ : for instance, we show that $\frac{\partial\phi}{\partial z_1}$ and $\frac{\partial\phi}{\partial z_2}$ always possess the same L^p -integrability on \mathbb{T}^2 , and we obtain combinatorial relations between intersection multiplicities at singularities and vanishing orders for branches of level sets. We also present several new methods of constructing rational inner functions that allow us to prescribe properties of their zero sets, unimodular level sets, and singularities.

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