A uniqueness result for functions with zero fine gradient on quasiconnected and finely connected sets

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Abstract. We show that every Sobolev function in $W^{1,p}_{\mathrm{loc}}(U)$ on a p-quasiopen set $U \subset \mathbb{R}^n$ with a.e.-vanishing p-fine gradient is a.e.-constant if and only if U is p-quasiconnected. To prove this we use the theory of Newtonian Sobolev spaces on metric measure spaces, and obtain the corresponding equivalence also for complete metric spaces equipped with a doubling measure supporting a p-Poincaré inequality. On unweighted $\mathbb{R}^n$, we also obtain the corresponding result for p-finely open sets in terms of p-fine connectedness, using a deep result by Latvala.