# Optimizing the first Dirichlet eigenvalue
# of the Laplacian with an obstacle

ANTOINE HENROT AND DAVIDE ZUCCO

**Abstract.** Inside a fixed bounded domain $\Omega$ of the plane, we look for the best compact connected set $K$, of given perimeter, in order to *maximize* the first Dirichlet eigenvalue $\lambda_1(\Omega \setminus K)$. We discuss some of the qualitative properties of the maximizers, moving toward existence, regularity, and geometry. Then we study the problem in specific domains: disks, rings, and, more generally, disks with convex holes. In these situations, we prove symmetry and in some cases non symmetry results, identifying the solution.

We choose to work with the *outer Minkowski content* as the "good" notion of perimeter. Therefore, we are led to prove some new properties for it as its lower semicontinuity with respect to the Hausdorff convergence and the fact that the outer Minkowski content is equal to the Hausdorff lower semicontinuous envelope of the classical perimeter.

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