On the slope conjecture of Barja and Stoppino for fibred surfaces

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Abstract. Let $f: X \to B$ be a locally non-trivial relatively minimal fibration of genus $g \ge 2$ with relative irregularity q_f . It was conjectured by Barja and Stoppino that the slope $\lambda_f \ge \frac{4(g-1)}{g-q_f}$. On the one hand, we show the lower bound $\lambda_f > \frac{4(g-1)}{g-q_f/2}$, and also prove the Barja-Stoppino conjecture when q_f is small with respect to g. On the other hand, we construct counterexamples violating the conjectured bound when g is odd and $q_f = (g+1)/2$.

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